# Lesson 7: Classification of Solutions

## Classwork

### Exercises

Solve each of the following equations for $x$.

1. $7x - 3 = 5x + 5$

$-5x \quad -5x$

$2x - 3 = 5$

$+3 \quad +3$

$\frac{2x}{2} = \frac{8}{2}$

$x = 4$

$\boxed{\text{One Solution}}$

$7(4) - 3 = 5(4) + 5$

$28 - 3 = 20 + 5$

$25 = 25$

weird

2. $7x - 3 = 7x + 5$

$-7x \quad -7x$

$-3 \neq 5$

$\boxed{\text{No Solution}}$ : same coefficient, different constants

* There exists no number that can ever be plugged in for the variable that will make this equation true.

3. $7x - 3 = -3 + 7x$

$-7x \quad -7x$

$-3 = -3$

$\boxed{\text{Infinitely Many Solutions}}$ : Same coefficients, Same constants

* Every number in existence will make this equation true, because the variable will always cancel out and leave equal constants.

one

$$\underline{5}x + 3 = \underline{4}x - 1$$

$$\underline{3}x - 5 = \underline{3}x + 2$$

$$-5 + 7x = 7x - 5$$

Many

One solution

$16 = 16$

$3x + 1 = 1x + 11$ given

$-1x \qquad -1x$

$2x + 1 = 11$

Give a brief explanation as to what kind of solution(s) you expect the following linear equations to have. Transform the equation into a simpler form if necessary.

4. $11x - 2x + 15 = 8 + 7 + 9x$

5. $3(x - 14) + 1 = -4x + 5$

6. $-3x + 32 - 7x = -2(5x + 10)$

7. $\frac{1}{2}(8x + 26) = 13 + 4x$

8. Write two equations that have no solutions.

$1{,}342x - 3 = 1{,}342x - 5$

$100x - 4 = -7 + 100x$

9. Write two equations that have one unique solution each.

$11x - 2 = 7x + 4$

$3x - 5 = 4x - 3$

10. Write two equations that have infinitely many solutions.

$4x - 2 = -2 + 4x$

$450x - 5 = -5 + 450x$

Many solutions

No Solution

Lesson Summary

There are three classifications of solutions to linear equations: one solution (unique solution), no solution, or infinitely many solutions.

Equations with no solution will, after being simplified, have coefficients of $x$ that are the same on both sides of the equal sign and constants that are different. For example, $x + b = x + c$, where $b$ and $c$ are constants that are not equal. A numeric example is $8x + 5 = 8x - 3$.

Equations with infinitely many solutions will, after being simplified, have coefficients of $x$ and constants that are the same on both sides of the equal sign. For example, $x + a = x + a$, where $a$ is a constant. A numeric example is $6x + 1 = 1 + 6x$.

## Problem Set

1. Give a brief explanation as to what kind of solution(s) you expect for the linear equation $18x + \frac{1}{2} = 6(3x + 25)$. Transform the equation into a simpler form if necessary.

2. Give a brief explanation as to what kind of solution(s) you expect for the linear equation $8 - 9x = 15x + 7 + 3x$. Transform the equation into a simpler form if necessary.

3. Give a brief explanation as to what kind of solution(s) you expect for the linear equation $5(x + 9) = 5x + 45$. Transform the equation into a simpler form if necessary.

4. Give three examples of equations where the solution will be unique, that is, only one solution is possible.

5. Solve one of the equations you wrote in Problem 4, and explain why it is the only solution.

6. Give three examples of equations where there will be no solution.

7. Attempt to solve one of the equations you wrote in Problem 6, and explain why it has no solution.

8. Give three examples of equations where there will be infinitely many solutions.

9. Attempt to solve one of the equations you wrote in Problem 8, and explain why it has infinitely many solutions.