Lesson 27: Nature of Solutions of a System of Linear Equations

Classwork

Exercises

Determine the nature of the solution to each system of linear equations.

1. $\begin{cases} 3x + 4y = 5 \\ y = -\frac{3}{4}x + 1 \end{cases}$

2. $\begin{cases} 7x + 2y = -4 \\ x - y = 5 \end{cases}$

3. $\begin{cases} 9x + 6y = 3 \\ 3x + 2y = 1 \end{cases}$

Fraction Rules: + / − common denominators; × straight across; ÷ multiply by the reciprocal

Determine the nature of the solution to each system of linear equations. If the system has a solution, find it algebraically, and then verify that your solution is correct by graphing.

4. $\begin{cases} y = 2x - 5 \\ 2y = -3x - 1 \end{cases}$ $\left(\frac{9}{7}, -\frac{17}{7}\right)$

$2(2x-5) = -3x - 1$

$4x - 10 = -3x - 1$

$7x - 10 = -1$

$7x = 9$

$x = \frac{9}{7}$

$y = 2\left(\frac{9}{7}\right) - 5$

$y = \frac{18}{7} - \frac{35}{7}$

$y = \frac{-17}{7}$

5. $\begin{cases} y = \frac{3}{2}x - 1 \\ 3y = x + 2 \end{cases}$ $\left(\frac{10}{7}, \frac{8}{7}\right)$

$3\left(\frac{3}{2}x - 1\right) = x + 2$

$\frac{9}{2}x - 3 = x + 2$

$-\frac{2}{2}x$

$\frac{7}{2}x - 3 = 2$

$+3 \quad +3$

$\frac{2}{7} \cdot \frac{7}{2}x = 5 \cdot \frac{2}{7}$

$x = \frac{10}{7}$

$y = \frac{3}{2}x - 1$

$y = \frac{3}{2}\left(\frac{10}{7}\right) - 1$

$y = \frac{30}{14} - \frac{14}{14}$

$y = \frac{16 \div 2}{14 \div 2}$

$y = \frac{8}{7}$

$\frac{2 \cdot 14}{1 \cdot 14}$ $\frac{28}{14}$ $14 \times 2 = 28$

6. $\begin{cases} x = 12y - 4 \\ x = 9y + 7 \end{cases}$

7. Write a system of equations with $(4, -5)$ as its solution.

Lesson Summary

A system of linear equations can have a unique solution, no solutions, or infinitely many solutions.

Systems with a unique solution will be comprised of linear equations that have different slopes that graph as distinct lines, intersecting at only one point.

Systems with no solution will be comprised of linear equations that have the same slope that graph as parallel lines (no intersection).

Systems with infinitely many solutions will be comprised of linear equations that have the same slope and y-intercept that graph as the same line. When equations graph as the same line, every solution to one equation will also be a solution to the other equation.

A system of linear equations can be solved using a substitution method. That is, if two expressions are equal to the same value, then they can be written equal to one another.

Example:

$$\begin{cases} y = 5x - 8 \\ y = 6x + 3 \end{cases}$$

Since both equations in the system are equal to y, we can write the equation, $5x - 8 = 6x + 3$, and use it to solve for x and then the system.

Example:

$$\begin{cases} 3x = 4y + 2 \\ x = y + 5 \end{cases}$$

Multiply each term of the equation $x = y + 5$ by 3 to produce the equivalent equation $3x = 3y + 15$. As in the previous example, since both equations equal $3x$, we can write $4y + 2 = 3y + 15$. This equation can be used to solve for y and then the system.

Problem Set

Determine the nature of the solution to each system of linear equations. If the system has a solution, find it algebraically, and then verify that your solution is correct by graphing.

1. $\begin{cases} y = \frac{3}{7}x - 8 \\ 3x - 7y = 1 \end{cases}$

2. $\begin{cases} 2x - 5 = y \\ -3x - 1 = 2y \end{cases}$

3. $\begin{cases} x = 6y + 7 \\ x = 10y + 2 \end{cases}$

4. $\begin{cases} 5y = \frac{15}{4}x + 25 \\ y = \frac{3}{4}x + 5 \end{cases}$

5. $\begin{cases} x + 9 = y \\ x = 4y - 6 \end{cases}$

6. $\begin{cases} 3y = 5x - 15 \\ 3y = 13x - 2 \end{cases}$

7. $\begin{cases} 6x - 7y = \frac{1}{2} \\ 12x - 14y = 1 \end{cases}$

8. $\begin{cases} 5x - 2y = 6 \\ -10x + 4y = -14 \end{cases}$

9. $\begin{cases} y = \frac{3}{2}x - 6 \\ 2y = 7 - 4x \end{cases}$

10. $\begin{cases} 7x - 10 = y \\ y = 5x + 12 \end{cases}$

11. Write a system of linear equations with $(-3, 9)$ as its solution.